boundary conditions. If we then introduce the same nondimensional variable transformations as before, we again obtain Eqs. (15–17) of the "canonical" triple-deck formulation with only the pressure relationship Eq. (14) generalized to

$$\hat{p} = \frac{K_T}{\gamma + 1}\left\{1 - \left[1 - \frac{3}{2}\frac{(\gamma + 1)}{K_T}\frac{\mathrm{d}}{\mathrm{d}\hat{x}}(\hat{y}_B + \hat{\delta}^*)\right]^{\frac{2}{3}}\right\} \tag{26}$$

where $K_T \equiv (M_\infty^2 - 1)^{\frac{5}{4}}/\varepsilon^2\lambda^{\frac{1}{2}}C_{\mathrm{REF}}^{\frac{1}{4}}$ is a transonic viscous interaction parameter. In the $K_T \gg 1$ limit, Eq. (26) reduces to Eq. (10) for higher Mach number supersonic flow.

Now, numerical studies of this modified transonic triple-deck problem have shown[15] that the value of $\hat{p}_{i.s.}$ changes very little over a wide range of K_T values: for $K_T \geq 1$, we may thus continue to use the incipient separation criterion Eq. (14). Then with $\mathrm{d}\hat{\delta}^*/\mathrm{d}\hat{x} = K_d\hat{\theta}_{i.s.}$, inversion of Eq. (22) followed by conversion back to physical variables yields

$$M_\infty\theta_{i.s.} \approx \frac{2(1 + K_d)^{-1}}{3(\gamma + 1)}M_\infty\beta^3\left\{1 - \left[1 - 1.03(\gamma + 1)\left(\frac{\sqrt{M_\infty^2 - 1}}{M_\infty}\chi C_{\mathrm{REF}}^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{\frac{3}{2}}\right\} \tag{27}$$

This is the desired extension of Eq. (19) to include the transonic regime. Although a bit complicated, it reveals the correct underlying scaling behavior while passing over to the simpler form of Eq. (19) for sufficiently small values of χ. In this regard we note that Eq. (27) is, in fact, a unified scaling law for the entire supersonic regime from transonic to moderately hypersonic.

Acknowledgments

The support of SERC Grant GRH26390 while the author was a visiting professor at Cranfield University College of Aeronautics and helpful discussions with J. L. Stollery of Cranfield are gratefully acknowledged.

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Simple Method of Supersonic Flow Visualization Using Watertable

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Introduction

THE direct shadowgraph technique is a relatively easy and common means for studying hydrodynamic phenomena in the flow of liquids. A number of visualization techniques for water flow currently exist, the use of which depend mainly on the desired test information, available facilities, and model flow speeds. An overview of the available flow visualization methods has been discussed by Werlé.[1] Another common way of visualizing the flow of liquids is to introduce optical disturbances in the liquid and to detect them by the Schlieren System[2] or the recently developed Sugar Schlieren System.[3] Flow visualization for liquid flow may be conducted in virtually any type of horizontal channel utilizing a smooth transparent bed of shallow water over a glass sheet.

Quantitative assessments of the behaviors of a flow from the recorded flow pattern obtained from conventional flow visualization techniques have often posed great difficulty to researchers. Yamamoto et al.[4,5] have developed an image processing technique from the flow pattern obtained in a free-surface watertable using inclined grid Moiré topography. Recently, a relatively simple and novel optical method for the quantitative evaluation of physical flow variables in a high-speed flow has been obtained from the photographs of the flow pattern obtained in a watertable using the established theory of hydraulic analogy.[6] This optical technique for processing the image patterns obtained photographically in a watertable seems to be new for the study of two-dimensional flowfield quantitatively. The present photographic method for flow visualization in a watertable has been demonstrated at supersonic speeds and may also be extended to study different model configurations.

Method and Procedure

The schematic arrangement of the experimental setup used to visualize the flowfield is illustrated in Fig. 1. Great care is necessary during the experiment to rid the flowfield of surging, turbulence, and angularity. A conventional 35-mm camera and black and white film are used to record the flow pattern. The picture of the flowfield obtained using this new technique as the flowfield moved past a symmetric airfoil with a sharp leading edge at a Froude number (corresponds to the Mach number in a gas flow) greater than unity is shown in Fig. 2. The flow is moving from top to bottom about the model. The introduction of the alternate bright and dark bands (marked as mechanical grating plate in Fig. 1) add much greater contrast to the image of the flow pattern which, incidentally, has a strong resemblance to the conventional interferogram. It differs from the conventional interferogram, however, for the resolution of the flowfield depends on the width, spacing, as well as the fineness of the parallel equidistant bands made on a transparent sheet.

The method of visualization involves superimposing the shadows of alternate bands on the flow placed in a plane at right angles to the flow direction. The position of the light sources used to illuminate the flowfield is important to get a better resolution of the image of

Received Sept. 27, 1993; revision received Jan. 7, 1994; accepted for publication March 4, 1994.

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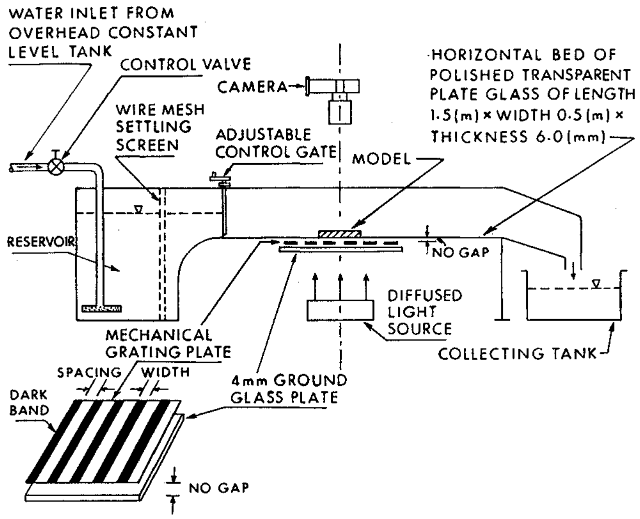

Fig. 1 Systematic arrangement of the experimental setup.

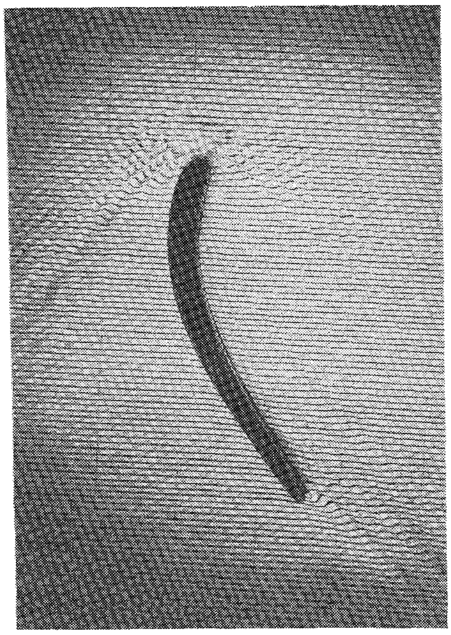

Fig. 2 Symmetric airfoil (maximum thickness 16 mm, chord length 144 mm) at Mach number 2.82.

the flowfield. The flow configuration (Fig. 2) with the background of mechanical grating is photographed at a shutter speed that has been moved down to 1/60. The exposure time and aperture (f number) which vary with the film quality and flow conditions are determined using a photoelectric exposure meter. A range of film speed is examined for use; however, 125 ASA film is found to be suitable and the positive images are carefully printed on high-contrast glossy photographic paper.

The dark bands in the grating plate, when viewed through the shock wave attached to the sharp leading edge of the airfoil, produce a distorted image of the regular band (Fig. 2). An earlier study[6] has shown that there is correspondence between the image of a dark band formed in the watertable and the image of an identical dark band seen when the watertable is viewed through an optical prism with its axis placed along the wave front (Fig. 2 in Ref. 6). The angular deviation of the dark bands at any point of the flowfield with respect to the undistorted or rather undisturbed band could be used as a direct measure of the flow characteristics (Figs. 4 and 5 in Ref. 6). This new image processing technique may be extended to determine quantitative parameters of the flowfield as shown in Fig. 2, particularly along the wave front where the distortion of the regular band is prominent.

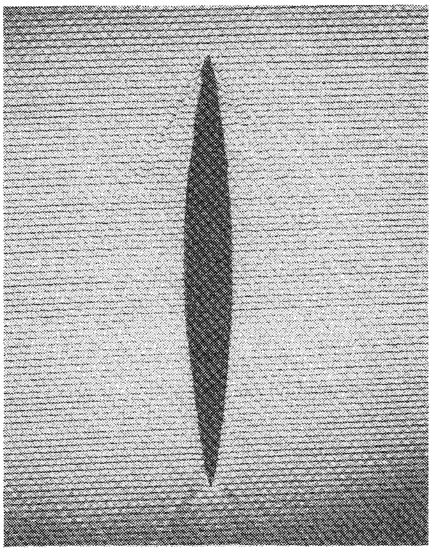

Fig. 3 Cambered airfoil at Mach number 1.86.

Case Problem

Figure 3 shows the flowfield (at $M > 1$) in the watertable about a cambered airfoil placed at an angle of incidence of 8 deg. The chord length and the maximum thickness of the airfoil are 126 mm and 10 mm, respectively. The detached bow shock in front of the blunt nose of the leading edge is clearly indicated by the abrupt deflection of the band with respect to the undisturbed one. The attached shock adjacent to the trailing edge is also quite prominent. A similar picture is difficult to obtain in a conventional wind tunnel using a smoke wire visualization method.[7] The flowfields in the expansion regions (downstream of the model) as indicated in the photographs (Figs. 2 and 3) are nearly potential as expected.

Conclusions

From the preceding study following points are evident:

1) The method used is very simple to implement on the watertable. The visual quality of the flowfield is quite comparable to, if not better than, the commonly available visualization techniques.

2) The overall cost of the experimental setup is relatively small in comparison with that of compressed air systems. Furthermore, the primary cost and time involved in recording the flow patterns are practically insignificant compared to those involved in the conventional techniques.

Quantitative analysis of the flow patterns using the optical prism method[6] may also add significantly to the capability of the existing image processing technique and open up entirely new application areas that have not previously been feasible or cost effective. In view of the potentiality of the proposed method, computerization of the photographic technique would be worth trying.

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